Package 'genridge'

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Type Package

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Imports rgl

Suggests MASS, bestglm, vcdExtra

Description The genridge package introduces generalizations of the standard univariate ridge trace plot used in ridge regression and related methods. These graphical methods show both bias (actually, shrinkage) and precision, by plotting the covariance ellipsoids of the estimated

coefficients, rather than just the estimates themselves. 2D and 3D plotting methods are provided, both in the space of the predictor variables and in the transformed space of the PCA/SVD of the predictors.

License GPL (>= 2)

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2 genridge-package

R topics documented:

	genridge-package																											2
	Acetylene																											3
	biplot.pcaridge .																											4
	contourf																											7
	Detroit																											8
	Manpower																											10
	pairs.ridge																											
	pca																											
	plot.ridge																											
	plot3d																											
	precision																											
	prostate																											
	ridge																											
	traceplot																											
	trans.colors																											
	vif.ridge																											
Index																												33
genri	idge-package	G	Gen	era	ıliz	;ed	! ri	dg	e ti	ra	ce	pi	lot	s f	or	ri	dg	e 1	eg	res	ssi	on	<u> </u>					

Description

The genridge package introduces generalizations of the standard univariate ridge trace plot used in ridge regression and related methods (Friendly, 2012). These graphical methods show both bias (actually, shrinkage) and precision, by plotting the covariance ellipsoids of the estimated coefficients, rather than just the estimates themselves. 2D and 3D plotting methods are provided, both in the space of the predictor variables and in the transformed space of the PCA/SVD of the predictors.

Details

This package provides computational support for the graphical methods described in Friendly (2013). Ridge regression models may be fit using the function ridge, which incorporates features of lm.ridge. In particular, the shrinkage factors in ridge regression may be specified either in terms of the constant added to the diagonal of X^TX matrix (lambda), or the equivalent number of degrees of freedom.

More importantly, the ridge function also calculates and returns the associated covariance matrices of each of the ridge estimates, allowing precision to be studied and displayed graphically.

This provides the support for the main plotting functions in the package:

plot.ridge: Bivariate ridge trace plots pairs.ridge: All pairwise bivariate ridge trace plots plot3d.ridge: 3D ridge trace plots traceplot: Traditional univariate ridge trace plots Acetylene 3

In addition, the function pca.ridge transforms the coefficients and covariance matrices of a ridge object from predictor space to the equivalent, but more interesting space of the PCA of X^TX or the SVD of X. The main plotting functions also work for these objects, of class c("ridge", "pcaridge").

Finally, the functions precision and vif.ridge provide other useful measures and plots.

Author(s)

Michael Friendly

Maintainer: Michael Friendly <friendly@yorku.ca>

References

Friendly, M. (2013). The Generalized Ridge Trace Plot: Visualizing Bias *and* Precision. *Journal of Computational and Graphical Statistics*, **22**(1), 50-68, doi:10.1080/10618600.2012.681237, https://www.datavis.ca/papers/genridge-jcgs.pdf

Arthur E. Hoerl and Robert W. Kennard (1970). Ridge Regression: Biased Estimation for Nonorthogonal Problems, *Technometrics*, 12(1), pp. 55-67.

Arthur E. Hoerl and Robert W. Kennard (1970). Ridge Regression: Applications to Nonorthogonal Problems *Technometrics*, 12(1), pp. 69-82.

See Also

lm.ridge

Examples

see examples for ridge, etc.

Acetylene

Acetylene Data

Description

The data consist of measures of yield of a chemical manufacturing process for acetylene in relation to numeric parameters.

Format

A data frame with 16 observations on the following 4 variables.

yield conversion percentage yield of acetylene temp reactor temperature (celsius) ratio H2 to N-heptone ratio time contact time (sec) 4 biplot.pcaridge

Details

Marquardt and Snee (1975) used these data to illustrate ridge regression in a model containing quadratic and interaction terms, particularly the need to center and standardize variables appearing in high-order terms.

Typical models for these data include the interaction of temp: ratio, and a squared term in temp

Source

SAS documentation example for PROC REG, Ridge Regression for Acetylene Data.

References

Marquardt, D.W., and Snee, R.D. (1975), "Ridge Regression in Practice," *The American Statistician*, **29**, 3-20.

Marquardt, D.W. (1980), "A Critique of Some Ridge Regression Methods: Comment," *Journal of the American Statistical Association*, Vol. 75, No. 369 (Mar., 1980), pp. 87-91

Examples

```
data(Acetylene)
# naive model, not using centering
amod0 <- lm(yield ~ temp + ratio + time + I(time^2) + temp:time, data=Acetylene)

y <- Acetylene[,"yield"]

X0 <- model.matrix(amod0)[,-1]

lambda <- c(0, 0.0005, 0.001, 0.002, 0.005, 0.01)
aridge0 <- ridge(y, X0, lambda=lambda)

traceplot(aridge0)
traceplot(aridge0, X="df")
pairs(aridge0, radius=0.2)</pre>
```

biplot.pcaridge

Biplot of Ridge Regression Trace Plot in SVD Space

Description

biplot.pcaridge supplements the standard display of the covariance ellipsoids for a ridge regression problem in PCA/SVD space with labeled arrows showing the contributions of the original variables to the dimensions plotted.

biplot.pcaridge 5

Usage

```
## S3 method for class 'pcaridge'
biplot(
 Х,
 variables = (p - 1):p,
 labels = NULL,
 asp = 1,
 origin,
 scale,
 var.lab = rownames(V),
 var.lwd = 1,
 var.col = "black",
 var.cex = 1,
 xlab,
 ylab,
 prefix = "Dim ",
 suffix = TRUE,
)
```

Arguments

X	A pcaridge object computed by pca.ridge or a ridge object.
variables	The dimensions or variables to be shown in the the plot. By default, the <i>last</i> two dimensions, corresponding to the smallest singular values, are plotted for class("pcaridge") objects or the <i>first</i> two variables for class("ridge") objects.
labels	A vector of character strings or expressions used as labels for the ellipses. Use labels=NULL to suppress these.
asp	Aspect ratio for the plot. The default value, asp=1 helps ensure that lengths and angles are preserved in these plots. Use asp=NA to override this.
origin	The origin for the variable vectors in this plot, a vector of length 2. If not specified, the function calculates an origin to make the variable vectors approximately centered in the plot window.
scale	The scale factor for variable vectors in this plot. If not specified, the function calculates a scale factor to make the variable vectors approximately fill the plot window.
var.lab	Labels for variable vectors. The default is the names of the predictor variables.
var.lwd, var.co	ol, var.cex
	Line width, color and character size used to draw and label the arrows representing the variables in this plot.
xlab, ylab	Labels for the plot dimensions. If not specified, prefix and suffix are used to construct informative dimension labels.
prefix	Prefix for labels of the plot dimensions.
suffix	Suffix for labels of the plot dimensions. If suffix=TRUE the percent of variance accounted for by each dimension is added to the axis label.

6 biplot.pcaridge

... Other arguments, passed to plot.pcaridge

Details

The biplot view showing the dimensions corresponding to the two *smallest* singular values is particularly useful for understanding how the predictors contribute to shrinkage in ridge regression.

This is only a biplot in the loose sense that results are shown in two spaces simultaneously – the transformed PCA/SVD space of the original predictors, and vectors representing the predictors projected into this space.

biplot.ridge is a similar extension of plot.ridge, adding vectors showing the relation of the PCA/SVD dimensions to the plotted variables.

class("ridge") objects use the transpose of the right singular vectors, t(x\$svd.V) for the dimension weights plotted as vectors.

Value

None

Author(s)

Michael Friendly, with contributions by Uwe Ligges

References

Friendly, M. (2013). The Generalized Ridge Trace Plot: Visualizing Bias *and* Precision. *Journal of Computational and Graphical Statistics*, **22**(1), 50-68, doi:10.1080/10618600.2012.681237, https://datavis.ca/papers/genridge-jcgs.pdf

See Also

```
plot.ridge, pca.ridge
```

```
longley.y <- longley[, "Employed"]
longley.X <- data.matrix(longley[, c(2:6,1)])

lambda <- c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(longley.y, longley.X, lambda=lambda)

plridge <- pca(lridge)

plot(plridge, radius=0.5)

# same, with variable vectors
biplot(plridge, radius=0.5)
# add some other options
biplot(plridge, radius=0.5, var.col="brown", var.lwd=2, var.cex=1.2, prefix="Dimension")</pre>
```

contourf 7

```
# biplots for ridge objects, showing PCA vectors
plot(lridge, radius=0.5)
biplot(lridge, radius=0.5, asp=NA)
```

contourf

Enhanced Contour Plots

Description

This is an enhancement to contour, written as a wrapper for that function. It creates a contour plot, or adds contour lines to an existing plot, allowing the contours to be filled and returning the list of contour lines.

Usage

```
contourf(
  x = seq(0, 1, length.out = nrow(z)),
  y = seq(0, 1, length.out = ncol(z)),
  z,
  nlevels = 10,
  levels = pretty(zlim, nlevels),
  zlim = range(z, finite = TRUE),
  col = par("fg"),
  color.palette = colorRampPalette(c("white", col)),
  fill.col = color.palette(nlevels + 1),
  fill.alpha = 0.5,
  add = FALSE,
  ...
)
```

Arguments

х, у	locations of grid lines at which the values in z are measured. These must be in ascending order. By default, equally spaced values from 0 to 1 are used. If x is a list, its components x\$x and x\$y are used for x and y, respectively. If the list has component x\$z this is used for z.
Z	a matrix containing the values to be plotted (NAs are allowed). Note that \boldsymbol{x} can be used instead of \boldsymbol{z} for convenience.
nlevels	number of contour levels desired iff levels is not supplied
levels	numeric vector of levels at which to draw contour lines
zlim	z-limits for the plot. x-limits and y-limits can be passed through
col	color for the lines drawn
color.palette	a color palette function to be used to assign fill colors in the plot

8 Detroit

fill.col	a call to the color.palette function or an an explicit set of colors to be used in the plot. Use fill.col=NULL to suppress the filled polygons. a vector of fill colors corresponding to levels. By default, a set of possibly transparent colors is calculated ranging from white to col, using transparency given by fill.alpha
fill.alpha	transparency value for fill.col, either a hex character string, or a numeric value between 0 and 1. Use fill.alpha=NA to suppress transparency.
add	logical. If TRUE, add to a current plot.
•••	additional arguments passed to contour, including all arguments of contour.default not mentioned above, as well as additional graphical parameters passed by contour.default to more basic functions.

Value

Returns invisibly the list of contours lines, with components levels, x, y. See contourLines.

Author(s)

Michael Friendly

See Also

```
contour, contourLines
contourplot from package lattice.
```

```
x <- 10*1:nrow(volcano)
y <- 10*1:ncol(volcano)
contourf(x,y,volcano, col="blue")
contourf(x,y,volcano, col="blue", nlevels=6)

# return value, unfilled, other graphic parameters
res <- contourf(x,y,volcano, col="blue", fill.col=NULL, lwd=2)
# levels used in the plot
sapply(res, function(x) x[[1]])</pre>
```

Detroit 9

Description

The data set Detroit was used extensively in the book by Miller (2002) on subset regression. The data are unusual in that a subset of three predictors can be found which gives a very much better fit to the data than the subsets found from the Efroymson stepwise algorithm, or from forward selection or backward elimination. They are also unusual in that, as time series data, the assumption of independence is patently violated, and the data suffer from problems of high collinearity.

As well, ridge regression reveals somewhat paradoxical paths of shrinkage in univariate ridge trace plots, that are more comprehensible in multivariate views.

Format

A data frame with 13 observations on the following 14 variables.

Police Full-time police per 100,000 population

Unemp Percent unemployed in the population

MfgWrk Number of manufacturing workers in thousands

GunLic Number of handgun licences per 100,000 population

GunReg Number of handgun registrations per 100,000 population

HClear Percent of homicides cleared by arrests

WhMale Number of white males in the population

NmfgWrk Number of non-manufacturing workers in thousands

GovWrk Number of government workers in thousands

HrEarn Average hourly earnings

WkEarn Average weekly earnings

Accident Death rate in accidents per 100,000 population

Assaults Number of assaults per 100,000 population

Homicide Number of homicides per 100,000 of population

Details

The data were originally collected and discussed by Fisher (1976) but the complete dataset first appeared in Gunst and Mason (1980, Appendix A). Miller (2002) discusses this dataset throughout his book, but doesn't state clearly which variables he used as predictors and which is the dependent variable. (Homicide was the dependent variable, and the predictors were Police ... WkEarn.) The data were obtained from StatLib.

A similar version of this data set, with different variable names appears in the bestglm package.

Source

http://lib.stat.cmu.edu/datasets/detroit

10 Manpower

References

Fisher, J.C. (1976). Homicide in Detroit: The Role of Firearms. Criminology, 14, 387–400.

Gunst, R.F. and Mason, R.L. (1980). Regression analysis and its application: A data-oriented approach. Marcel Dekker.

Miller, A. J. (2002). Subset Selection in Regression. 2nd Ed. Chapman & Hall/CRC. Boca Raton.

```
data(Detroit)
# Work with a subset of predictors, from Miller (2002, Table 3.14),
# the "best" 6 variable model
     Variables: Police, Unemp, GunLic, HClear, WhMale, WkEarn
# Scale these for comparison with other methods
Det \leftarrow as.data.frame(scale(Detroit[,c(1,2,4,6,7,11)]))
Det <- cbind(Det, Homicide=Detroit[,"Homicide"])</pre>
# use the formula interface; specify ridge constants in terms
# of equivalent degrees of freedom
dridge <- ridge(Homicide ~ ., data=Det, df=seq(6,4,-.5))</pre>
# univariate trace plots are seemingly paradoxical in that
# some coefficients "shrink" *away* from 0
traceplot(dridge, X="df")
vif(dridge)
pairs(dridge, radius=0.5)
plot3d(dridge, radius=0.5, labels=dridge$df)
# transform to PCA/SVD space
dpridge <- pca(dridge)</pre>
# not so paradoxical in PCA space
traceplot(dpridge, X="df")
biplot(dpridge, radius=0.5, labels=dpridge$df)
# show PCA vectors in variable space
biplot(dridge, radius=0.5, labels=dridge$df)
```

Manpower 11

Description

The hospital manpower data, taken from Myers (1990), table 3.8, are a well-known example of highly collinear data to which ridge regression and various shrinkage and selection methods are often applied.

The data consist of measures taken at 17 U.S. Naval Hospitals and the goal is to predict the required monthly man hours for staffing purposes.

Format

A data frame with 17 observations on the following 6 variables.

Hours monthly man hours (response variable)

Load average daily patient load

Xray monthly X-ray exposures

BedDays monthly occupied bed days

AreaPop eligible population in the area in thousands

Stay average length of patient's stay in days

Details

Myers (1990) indicates his source was "Procedures and Analysis for Staffing Standards Development: Data/Regression Analysis Handbook", Navy Manpower and Material Analysis Center, San Diego, 1979.

Source

Raymond H. Myers (1990). *Classical and Modern Regression with Applications*, 2nd ed., PWS-Kent, pp. 130-133.

References

Donald R. Jensen and Donald E. Ramirez (2012). Variations on Ridge Traces in Regression, *Communications in Statistics - Simulation and Computation*, 41 (2), 265-278.

See Also

manpower for the same data, and other analyses

```
data(Manpower)
mmod <- lm(Hours ~ ., data=Manpower)
vif(mmod)
# ridge regression models, specified in terms of equivalent df
mridge <- ridge(Hours ~ ., data=Manpower, df=seq(5, 3.75, -.25))
vif(mridge)</pre>
```

12 pairs.ridge

```
# univariate ridge trace plots
traceplot(mridge)
traceplot(mridge, X="df")

# bivariate ridge trace plots
plot(mridge, radius=0.25, labels=mridge$df)
pairs(mridge, radius=0.25)

# 3D views
# ellipsoids for Load, Xray & BedDays are nearly 2D
plot3d(mridge, radius=0.2, labels=mridge$df)
# variables in model selected by AIC & BIC
plot3d(mridge, variables=c(2,3,5), radius=0.2, labels=mridge$df)

# plots in PCA/SVD space
mpridge <- pca(mridge)
traceplot(mpridge, X="df")
biplot(mpridge, radius=0.25)</pre>
```

pairs.ridge

Scatterplot Matrix of Bivariate Ridge Trace Plots

Description

Displays all possible pairs of bivariate ridge trace plots for a given set of predictors.

Usage

pairs.ridge 13

Arguments

x	A ridge object, as fit by ridge
variables	Predictors in the model to be displayed in the plot: an integer or character vector, giving the indices or names of the variables.
radius	Radius of the ellipse-generating circle for the covariance ellipsoids.
lwd, lty	Line width and line type for the covariance ellipsoids. Recycled as necessary.
col	A numeric or character vector giving the colors used to plot the covariance ellipsoids. Recycled as necessary.
center.pch	Plotting character used to show the bivariate ridge estimates. Recycled as necessary.
center.cex	Size of the plotting character for the bivariate ridge estimates
digits	Number of digits to be displayed as the (min, max) values in the diagonal panels
diag.cex	Character size for predictor labels in diagonal panels
diag.panel	Function to draw diagonal panels. Not yet implemented: just uses internal panel.label to write the variable name and ranges.
fill	Logical vector: Should the covariance ellipsoids be filled? Recycled as necessary.
fill.alpha	Numeric vector: alpha transparency value(s) for filled ellipsoids. Recycled as necessary.
•••	Other arguments passed down

Value

None. Used for its side effect of plotting.

Author(s)

Michael Friendly

References

Friendly, M. (2013). The Generalized Ridge Trace Plot: Visualizing Bias *and* Precision. *Journal of Computational and Graphical Statistics*, **22**(1), 50-68, doi:10.1080/10618600.2012.681237, https://www.datavis.ca/papers/genridge-jcgs.pdf

See Also

ridge for details on ridge regression as implemented here plot.ridge, traceplot for other plotting methods

14 pca

Examples

```
longley.y <- longley[, "Employed"]
longley.X <- data.matrix(longley[, c(2:6,1)])

lambda <- c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(longley.y, longley.X, lambda=lambda)

pairs(lridge, radius=0.5, diag.cex=1.75)

data(prostate)
py <- prostate[, "lpsa"]
pX <- data.matrix(prostate[, 1:8])
pridge <- ridge(py, pX, df=8:1)

pairs(pridge)</pre>
```

рса

Transform Ridge Estimates to PCA Space

Description

The function pca.ridge transforms a ridge object from parameter space, where the estimated coefficients are β_k with covariance matrices Σ_k , to the principal component space defined by the right singular vectors, V, of the singular value decomposition of the scaled predictor matrix, X.

In this space, the transformed coefficients are $V\beta_k$, with covariance matrices

$$V\Sigma_k V^T$$

.

This transformation provides alternative views of ridge estimates in low-rank approximations. In particular, it allows one to see where the effects of collinearity typically reside — in the smallest PCA dimensions.

Usage

```
pca(x, ...)
```

Arguments

x A ridge object, as fit by ridge

... Other arguments passed down. Not presently used in this implementation.

Value

An object of class c("ridge", "pcaridge"), with the same components as the original ridge object.

plot.ridge 15

Author(s)

Michael Friendly

References

Friendly, M. (2013). The Generalized Ridge Trace Plot: Visualizing Bias *and* Precision. *Journal of Computational and Graphical Statistics*, **22**(1), 50-68, doi:10.1080/10618600.2012.681237, https://www.datavis.ca/papers/genridge-jcgs.pdf

See Also

ridge

Examples

```
longley.y <- longley[, "Employed"]
longley.X <- data.matrix(longley[, c(2:6,1)])

lambda <- c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(longley.y, longley.X, lambda=lambda)

plridge <- pca(lridge)
traceplot(plridge)
pairs(plridge)
# view in space of smallest singular values
plot(plridge, variables=5:6)</pre>
```

plot.ridge

Bivariate Ridge Trace Plots

Description

The bivariate ridge trace plot displays 2D projections of the covariance ellipsoids for a set of ridge regression estimates indexed by a ridge tuning constant.

The centers of these ellipses show the bias induced for each parameter, and also how the change in the ridge estimate for one parameter is related to changes for other parameters.

The size and shapes of the covariance ellipses show directly the effect on precision of the estimates as a function of the ridge tuning constant.

16 plot.ridge

Usage

```
## S3 method for class 'ridge'
plot(
 х,
 variables = 1:2,
 radius = 1,
 which.lambda = 1:length(x$lambda),
 labels = lambda,
 pos = 3,
 cex = 1.2,
 1wd = 2,
 lty = 1,
 xlim,
 ylim,
  col = c("black", "red", "darkgreen", "blue", "darkcyan", "magenta", "brown",
    "darkgray"),
 center.pch = 16,
  center.cex = 1.5,
  fill = FALSE,
  fill.alpha = 0.3,
 ref = TRUE,
 ref.col = gray(0.7),
)
```

Arguments

x	A ridge object, as fit by ridge
variables	Predictors in the model to be displayed in the plot: an integer or character vector of length 2, giving the indices or names of the variables. Defaults to the first two predictors for ridge objects or the <i>last</i> two dimensions for pcaridge objects.
radius	Radius of the ellipse-generating circle for the covariance ellipsoids. The default, radius=1 gives a standard "unit" ellipsoid. Typically, values radius<1 gives less cluttered displays.
which.lambda	A vector of indices used to select the values of lambda for which ellipses are plotted. The default is to plot ellipses for all values of lambda in the ridge object.
labels	A vector of character strings or expressions used as labels for the ellipses. Use labels=NULL to suppress these.
pos, cex	Scalars or vectors of positions (relative to the ellipse centers) and character size used to label the ellipses
lwd, lty	Line width and line type for the covariance ellipsoids. Recycled as necessary.
xlim, ylim	X, Y limits for the plot, each a vector of length 2. If missing, the range of the covariance ellipsoids is used.
col	A numeric or character vector giving the colors used to plot the covariance ellipsoids. Recycled as necessary.

plot.ridge 17

center.pch	Plotting character used to show the bivariate ridge estimates. Recycled as necessary.
center.cex	Size of the plotting character for the bivariate ridge estimates
fill	Logical vector: Should the covariance ellipsoids be filled? Recycled as necessary.
fill.alpha	Numeric vector: alpha transparency value(s) in the range (0, 1) for filled ellipsoids. Recycled as necessary.
ref	Logical: whether to draw horizontal and vertical reference lines at 0.
ref.col	Color of reference lines.
• • •	Other arguments passed down to plot.default, e.g., xlab, ylab, and other graphic parameters.

Value

None. Used for its side effect of plotting.

Author(s)

Michael Friendly

References

Friendly, M. (2013). The Generalized Ridge Trace Plot: Visualizing Bias *and* Precision. *Journal of Computational and Graphical Statistics*, **22**(1), 50-68, doi:10.1080/10618600.2012.681237, https://www.datavis.ca/papers/genridge-jcgs.pdf

See Also

```
ridge for details on ridge regression as implemented here pairs.ridge, traceplot, biplot.pcaridge and plot3d.ridge for other plotting methods
```

18 plot3d

```
data(prostate)
py <- prostate[, "lpsa"]
pX <- data.matrix(prostate[, 1:8])
pridge <- ridge(py, pX, df=8:1)

plot(pridge)
plot(pridge, fill=c(TRUE, rep(FALSE,7)))</pre>
```

plot3d

3D Ridge Trace Plots

Description

The 3D ridge trace plot displays 3D projections of the covariance ellipsoids for a set of ridge regression estimates indexed by a ridge tuning constant.

The centers of these ellipses show the bias induced for each parameter, and also how the change in the ridge estimate for one parameter is related to changes for other parameters.

The size and shapes of the covariance ellipsoids show directly the effect on precision of the estimates as a function of the ridge tuning constant.

plot3d.ridge and plot3d.pcaridge differ only in the defaults for the variables plotted.

Usage

```
plot3d(x, ...)
## S3 method for class 'pcaridge'
plot3d(x, variables = (p - 2):p, ...)
## S3 method for class 'ridge'
plot3d(
  Х,
  variables = 1:3,
  radius = 1,
  which.lambda = 1:length(x$lambda),
  lwd = 1,
  lty = 1,
  xlim,
  ylim,
  zlim,
  xlab,
  ylab,
  zlab,
  col = c("black", "red", "darkgreen", "blue", "darkcyan", "magenta", "brown",
    "darkgray"),
```

plot3d

```
labels = lambda,
ref = TRUE,
ref.col = gray(0.7),
segments = 40,
shade = TRUE,
shade.alpha = 0.1,
wire = FALSE,
aspect = 1,
add = FALSE,
...
)
```

Arguments

x A ridge object, as fit by ridge or a pcaridge object as transformed by pca.ridge

... Other arguments passed down

variables Predictors in the model to be displayed in the plot: an integer or character vector

of length 3, giving the indices or names of the variables. Defaults to the first three predictors for ridge objects or the last three dimensions for pcaridge

objects.

radius Radius of the ellipse-generating circle for the covariance ellipsoids. The default,

radius=1 gives a standard "unit" ellipsoid. Typically, radius<1 gives less clut-

tered displays.

which.lambda A vector of indices used to select the values of lambda for which ellipsoids are

plotted. The default is to plot ellipsoids for all values of lambda in the ridge

object.

lwd, 1ty Line width and line type for the covariance ellipsoids. Recycled as necessary.

xlim, ylim, zlim

X, Y, Z limits for the plot, each a vector of length 2. If missing, the range of the

covariance ellipsoids is used.

xlab, ylab, zlab

Labels for the X, Y, Z variables in the plot. If missing, the names of the predic-

tors given in variables is used.

col A numeric or character vector giving the colors used to plot the covariance el-

lipsoids. Recycled as necessary.

labels A numeric or character vector giving the labels to be drawn at the centers of the

covariance ellipsoids.

ref Logical: whether to draw horizontal and vertical reference lines at 0. This is not

yet implemented.

ref.col Color of reference lines.

segments Number of line segments used in drawing each dimension of a covariance ellip-

soid

shade a logical scalar or vector, indicating whether the ellipsoids should be rendered

with shade3d. Recycled as necessary.

20 plot3d

shade.alpha a numeric value in the range [0,1], or a vector of such values, giving the alpha transparency for ellipsoids rendered with shade=TRUE.

a logical scalar or vector, indicating whether the ellipsoids should be rendered

with wire3d. Recycled as necessary.

aspect a scalar or vector of length 3, or the character string "iso", indicating the ratios

of the x, y, and z axes of the bounding box. The default, aspect=1 makes the bounding box display as a cube approximately filling the display. See aspect3d

for details.

add if TRUE, add to the current rgl plot; the default is FALSE.

Value

wire

None. Used for its side-effect of plotting

Note

This is an initial implementation. The details and arguments are subject to change.

Author(s)

Michael Friendly

References

Friendly, M. (2013). The Generalized Ridge Trace Plot: Visualizing Bias *and* Precision. *Journal of Computational and Graphical Statistics*, **22**(1), 50-68, doi:10.1080/10618600.2012.681237, https://www.datavis.ca/papers/genridge-jcgs.pdf

See Also

```
plot.ridge, pairs.ridge, pca.ridge
```

precision 21

rec		

Measures of Precision and Shrinkage for Ridge Regression

Description

Calculates measures of precision based on the size of the estimated covariance matrices of the parameters and shrinkage of the parameters in a ridge regression model. function does. ~~

Three measures of (inverse) precision based on the "size" of the covariance matrix of the parameters are calculated. Let V_k be the covariance matrix for a given ridge constant, and let λ_i , $i=1,\ldots p$ be its eigenvalues

- 1. $\log |V_k| = \log \prod \lambda$ or $|V_k|^{1/p} = (\prod \lambda)^{1/p}$ measures the linearized volume of the covariance ellipsoid and corresponds conceptually to Wilks' Lambda criterion
- 2. $trace(V_k) = \sum \lambda$ corresponds conceptually to Pillai's trace criterion
- 3. $\lambda_1 = max(\lambda)$ corresponds to Roy's largest root criterion.

Usage

```
precision(object, det.fun, normalize, ...)
```

Arguments

object	An object of class ridge or lm
det.fun	Function to be applied to the determinants of the covariance matrices, one of $c("log","root")$.
normalize	If TRUE the length of the coefficient vector is normalized to a maximum of 1.0.
	Other arguments (currently unused)

Value

A data.frame with the following columns

lambda	The ridge constant
df	The equivalent effective degrees of freedom
det	The det. fun function of the determinant of the covariance matrix
trace	The trace of the covariance matrix
max.eig	Maximum eigen value of the covariance matrix
norm.beta	The root mean square of the estimated coefficients, possibly normalized

22 precision

Note

Models fit by lm and ridge use a different scaling for the predictors, so the results of precision for an lm model will not correspond to those for ridge with ridge constant = 0.

Author(s)

Michael Friendly

See Also

ridge,

```
longley.y <- longley[, "Employed"]</pre>
longley.X <- data.matrix(longley[, c(2:6,1)])</pre>
lambda \leftarrow c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(longley.y, longley.X, lambda=lambda)</pre>
clr <- c("black", rainbow(length(lambda)-1, start=.6, end=.1))</pre>
coef(lridge)
(pdat <- precision(lridge))</pre>
# plot log |Var(b)| vs. length(beta)
with(pdat, {
plot(norm.beta, det, type="b",
cex.lab=1.25, pch=16, cex=1.5, col=clr, lwd=2,
xlab='shrinkage: ||b|| / max(||b||)',
ylab='variance: log |Var(b)|')
text(norm.beta, det, lambda, cex=1.25, pos=c(rep(2,length(lambda)-1),4))
text(min(norm.beta), max(det), "Variance vs. Shrinkage", cex=1.5, pos=4)
})
# plot trace[Var(b)] vs. length(beta)
with(pdat, {
plot(norm.beta, trace, type="b",
cex.lab=1.25, pch=16, cex=1.5, col=clr, lwd=2,
xlab='shrinkage: ||b|| / max(||b||)',
ylab='variance: trace [Var(b)]')
text(norm.beta, trace, lambda, cex=1.25, pos=c(2, rep(4,length(lambda)-1)))
# text(min(norm.beta), max(det), "Variance vs. Shrinkage", cex=1.5, pos=4)
})
```

prostate 23

prostate

Prostate Cancer Data

Description

Data to examine the correlation between the level of prostate-specific antigen and a number of clinical measures in men who were about to receive a radical prostatectomy.

Format

A data frame with 97 observations on the following 10 variables.

lcavol log cancer volume

lweight log prostate weight

age in years

lbph log of the amount of benign prostatic hyperplasia

svi seminal vesicle invasion

lcp log of capsular penetration

gleason a numeric vector

pgg45 percent of Gleason score 4 or 5

lpsa response

train a logical vector

Details

This data set came originally from the (now defunct) ElemStatLearn package.

The last column indicates which 67 observations were used as the "training set" and which 30 as the test set, as described on page 48 in the book.

Note

There was an error in this dataset in earlier versions of the package, as indicated in a footnote on page 3 of the second edition of the book. As of version 2012.04-0 this was corrected.

Source

Stamey, T., Kabalin, J., McNeal, J., Johnstone, I., Freiha, F., Redwine, E. and Yang, N (1989) Prostate specific antigen in the diagnosis and treatment of adenocarcinoma of the prostate II. Radical prostatectomy treated patients, *Journal of Urology*, **16**: 1076–1083.

24 ridge

Examples

```
data(prostate)
str( prostate )
cor( prostate[,1:8] )
prostate <- prostate[, -10]

prostate.mod <- lm(lpsa ~ ., data=prostate)
vif(prostate.mod)

py <- prostate[, "lpsa"]
pX <- data.matrix(prostate[, 1:8])
pridge <- ridge(py, pX, df=8:1)
pridge

plot(pridge)
pairs(pridge)
traceplot(pridge, X="df")</pre>
```

ridge

Ridge Regression Estimates

Description

The function ridge fits linear models by ridge regression, returning an object of class ridge designed to be used with the plotting methods in this package.

Ridge regression shrinkage can be parameterized in several ways. If a vector of lambda values is supplied, these are used directly in the ridge regression computations. Otherwise, if a vector df is supplied the equivalent values of lambda. In either case, both lambda and df are returned in the ridge object, but the rownames of the coefficients are given in terms of lambda.

Usage

```
ridge(y, ...)
## S3 method for class 'formula'
ridge(formula, data, lambda = 0, df, svd = TRUE, ...)
## Default S3 method:
ridge(y, X, lambda = 0, df, svd = TRUE, ...)
## S3 method for class 'ridge'
coef(object, ...)
```

ridge 25

```
## S3 method for class 'ridge'
print(x, digits = max(5, getOption("digits") - 5), ...)
## S3 method for class 'ridge'
vcov(object, ...)
```

Arguments

y A numeric vector containing the response variable. NAs not allowed.

. . . Other arguments, passed down to methods

formula For the formula method, a two-sided formula.

data For the formula method, data frame within which to evaluate the formula.

lambda A scalar or vector of ridge constants. A value of 0 corresponds to ordinary least

squares.

df A scalar or vector of effective degrees of freedom corresponding to lambda

svd If TRUE the SVD of the centered and scaled X matrix is returned in the ridge

object.

X A matrix of predictor variables. NA's not allowed. Should not include a column

of 1's for the intercept.

x, object An object of class ridge

digits For the print method, the number of digits to print.

Value

A list with the following components:

lambda The vector of ridge constants

df The vector of effective degrees of freedom corresponding to lambda

coef The matrix of estimated ridge regression coefficients

scales scalings used on the X matrix

kHKB HKB estimate of the ridge constant kLW L-W estimate of the ridge constant

GCV vector of GCV values

kGCV value of lambda with the minimum GCV

If svd==TRUE, the following are also included:

svd.D Singular values of the svd of the scaled X matrix

svd.U Left singular vectors of the svd of the scaled X matrix. Rows correspond to

observations and columns to dimensions.

svd. V Right singular vectors of the svd of the scaled X matrix. Rows correspond to

variables and columns to dimensions.

26 ridge

Author(s)

Michael Friendly

References

Hoerl, A. E., Kennard, R. W., and Baldwin, K. F. (1975), "Ridge Regression: Some Simulations," *Communications in Statistics*, 4, 105-123.

Lawless, J.F., and Wang, P. (1976), "A Simulation Study of Ridge and Other Regression Estimators," *Communications in Statistics*, 5, 307-323.

See Also

```
lm.ridge for other implementations of ridge regression
traceplot, plot.ridge, pairs.ridge, plot3d.ridge, for 1D, 2D, 3D plotting methods
pca.ridge, biplot.ridge, biplot.pcaridge for views in PCA/SVD space
precision.ridge for measures of shrinkage and precision
```

```
#\donttest{
# Longley data, using number Employed as response
longley.y <- longley[, "Employed"]</pre>
longley.X <- data.matrix(longley[, c(2:6,1)])</pre>
lambda \leftarrow c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(longley.y, longley.X, lambda=lambda)</pre>
# same, using formula interface
lridge <- ridge(Employed ~ GNP + Unemployed + Armed.Forces + Population + Year + GNP.deflator,</pre>
data=longley, lambda=lambda)
coef(lridge)
# standard trace plot
traceplot(lridge)
# plot vs. equivalent df
traceplot(lridge, X="df")
pairs(lridge, radius=0.5)
#}
data(prostate)
py <- prostate[, "lpsa"]</pre>
pX <- data.matrix(prostate[, 1:8])</pre>
pridge <- ridge(py, pX, df=8:1)</pre>
pridge
```

traceplot 27

```
plot(pridge)
pairs(pridge)
traceplot(pridge)
traceplot(pridge, X="df")
# Hospital manpower data from Table 3.8 of Myers (1990)
data(Manpower)
str(Manpower)
mmod <- lm(Hours ~., data=Manpower)
vif(mmod)
# ridge regression models, specified in terms of equivalent df
mridge <- ridge(Hours ~ ., data=Manpower, df=seq(5, 3.75, -.25))</pre>
vif(mridge)
# univariate ridge trace plots
traceplot(mridge)
traceplot(mridge, X="df")
# bivariate ridge trace plots
plot(mridge, radius=0.25, labels=mridge$df)
pairs(mridge, radius=0.25)
# 3D views
# ellipsoids for Load, Xray & BedDays are nearly 2D
plot3d(mridge, radius=0.2, labels=mridge$df)
# variables in model selected by AIC & BIC
plot3d(mridge, variables=c(2,3,5), radius=0.2, labels=mridge$df)
# plots in PCA/SVD space
mpridge <- pca(mridge)</pre>
traceplot(mpridge, X="df")
biplot(mpridge, radius=0.25)
```

traceplot

Univariate Ridge Trace Plots

Description

The traceplot function extends and simplifies the univariate ridge trace plots for ridge regression provided in the plot method for lm.ridge

Usage

```
traceplot(
```

28 traceplot

```
x,
X = c("lambda", "df"),
col = c("black", "red", "darkgreen", "blue", "darkcyan", "magenta", "brown",
    "darkgray"),
pch = c(15:18, 7, 9, 12, 13),
xlab,
ylab = "Coefficient",
xlim,
ylim,
...
)
```

Arguments

x	A ridge object, as fit by ridge
Χ	What to plot as the horizontal coordinate, one of c("lambda", "df")
col	A numeric or character vector giving the colors used to plot the ridge trace curves. Recycled as necessary.
pch	Vector of plotting characters used to plot the ridge trace curves. Recycled as necessary.
xlab	Label for horizontal axis
ylab	Label for vertical axis
xlim, ylim	x, y limits for the plot
	Other arguments passed to matplot

Details

For ease of interpretation, the variables are labeled at the side of the plot (left, right) where the coefficient estimates are expected to be most widely spread. If xlim is not specified, the range of the X variable is extended slightly to accommodate the variable names.

Value

None. Used for its side effect of plotting.

Author(s)

Michael Friendly

References

Friendly, M. (2013). The Generalized Ridge Trace Plot: Visualizing Bias *and* Precision. *Journal of Computational and Graphical Statistics*, **22**(1), 50-68, doi:10.1080/10618600.2012.681237, https://www.datavis.ca/papers/genridge-jcgs.pdf

Hoerl, A. E. and Kennard R. W. (1970). "Ridge Regression: Applications to Nonorthogonal Problems", *Technometrics*, 12(1), 69-82.

trans.colors 29

See Also

```
ridge for details on ridge regression as implemented here plot.ridge, pairs.ridge for other plotting methods
```

Examples

```
longley.y <- longley[, "Employed"]
longley.X <- data.matrix(longley[, c(2:6,1)])

lambda <- c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(longley.y, longley.X, lambda=lambda)

traceplot(lridge)
#abline(v=lridge$kLW, lty=3)
#abline(v=lridge$kHKB, lty=3)
#text(lridge$kHKB, -3, "LW")
#text(lridge$kHKB, -3, "HKB")

traceplot(lridge, X="df")</pre>
```

trans.colors

Make Colors Transparent

Description

Takes a vector of colors (as color names or rgb hex values) and adds a specified alpha transparency to each.

Usage

```
trans.colors(col, alpha = 0.5, names = NULL)
```

Arguments

col A character vector of colors, either as color names or rgb hex values

alpha alpha transparency value(s) to apply to each color (0 means fully transparent and

1 means opaque)

names optional character vector of names for the colors

Details

Colors (col) and alpha need not be of the same length. The shorter one is replicated to make them of the same length.

30 vif.ridge

Value

A vector of color values of the form "#rrggbbaa"

Author(s)

Michael Friendly

See Also

```
col2rgb, rgb,
```

Examples

```
trans.colors(palette(), alpha=0.5)

# alpha can be vectorized
trans.colors(palette(), alpha=seq(0, 1, length=length(palette())))

# lengths need not match: shorter one is repeated as necessary
trans.colors(palette(), alpha=c(.1, .2))

trans.colors(colors()[1:20])

# single color, with various alphas
trans.colors("red", alpha=seq(0,1, length=5))
# assign names
trans.colors("red", alpha=seq(0,1, length=5), names=paste("red", 1:5, sep=""))
```

vif.ridge

Variance Inflation Factors for Ridge Regression

Description

The function vif.ridge calculates variance inflation factors for the predictors in a set of ridge regression models indexed by the tuning/shrinkage factor.

Usage

```
## S3 method for class 'ridge'
vif(mod, ...)
```

Arguments

```
mod A ridge object
... Other arguments (unused)
```

vif.ridge 31

Details

Variance inflation factors are calculated using the simplified formulation in Fox & Monette (1992).

Value

Returns a matrix of variance inflation factors of the same size and shape as coef{mod}. The columns correspond to the predictors in the model and the rows correspond to the values of lambda in ridge estimation.

Author(s)

Michael Friendly

References

Fox, J. and Monette, G. (1992). Generalized collinearity diagnostics. JASA, 87, 178-183

See Also

```
vif, precision
```

```
data(longley)
lmod <- lm(Employed ~ GNP + Unemployed + Armed.Forces + Population +</pre>
                       Year + GNP.deflator, data=longley)
vif(lmod)
longley.y <- longley[, "Employed"]</pre>
longley.X <- data.matrix(longley[, c(2:6,1)])</pre>
lambda \leftarrow c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(longley.y, longley.X, lambda=lambda)</pre>
coef(lridge)
vridge <- vif(lridge)</pre>
vridge
# plot VIFs
pch <- c(15:18, 7, 9)
clr <- c("black", rainbow(5, start=.6, end=.1))</pre>
matplot(rownames(vridge), vridge, type='b',
xlab='Ridge constant (k)', ylab="Variance Inflation",
xlim=c(0, 0.08),
col=clr, pch=pch, cex=1.2)
text(0.0, vridge[1,], colnames(vridge), pos=4)
matplot(lridge$df, vridge, type='b',
```

32 vif.ridge

```
xlab='Degrees of freedom', ylab="Variance Inflation",
col=clr, pch=pch, cex=1.2)
text(6, vridge[1,], colnames(vridge), pos=2)

# more useful to plot VIF on the sqrt scale

matplot(rownames(vridge), sqrt(vridge), type='b',
xlab='Ridge constant (k)', ylab=expression(sqrt(VIF)),
xlim=c(-0.01, 0.08),
col=clr, pch=pch, cex=1.2, cex.lab=1.25)
text(-0.01, sqrt(vridge[1,]), colnames(vridge), pos=4, cex=1.2)

matplot(lridge$df, sqrt(vridge), type='b',
xlab='Degrees of freedom', ylab=expression(sqrt(VIF)),
col=clr, pch=pch, cex=1.2, cex.lab=1.25)
text(6, sqrt(vridge[1,]), colnames(vridge), pos=2, cex=1.2)
```

Index

* color	contourf, 7
trans.colors, 29	contourLines, 8
* datasets	contourplot, 8
Acetylene, 3	
Detroit, 8	Detroit, 8
Manpower, 10	
prostate, 23	genridge (genridge-package), 2
* dplot	genridge-package, 2
pca, 14	lm.ridge, 2, 3, 26, 27
* hplot	III.1 1dgc, 2, 3, 20, 27
biplot.pcaridge,4	Manpower, 10
contourf, 7	manpower, 11
pairs.ridge, 12	matplot, 28
plot.ridge, 15	,
plot3d, 18	pairs.ridge, 2, 12, 17, 20, 26, 29
traceplot, 27	pca, 14
* models	pca.ridge, 3, 5, 6, 19, 20, 26
precision, 21	plot.default, <i>17</i>
ridge, 24	plot.pcaridge, 6
vif.ridge, 30	<pre>plot.pcaridge(plot.ridge), 15</pre>
* multivariate	plot.ridge, 2, 6, 13, 15, 20, 26, 29
pca, 14	plot3d, 18
* package	plot3d.ridge, 2, 17, 26
genridge-package, 2	precision, <i>3</i> , 21, <i>31</i>
* regression	precision.ridge, 26
precision, 21	print.ridge (ridge), 24
ridge, 24	prostate, 23
vif.ridge, 30	
11.11.1460, 50	rgb, <i>30</i>
Acetylene, 3	ridge, 2, 13–17, 19, 22, 24, 28, 29
aspect3d, 20	
20,000,000	shade3d, <i>19</i>
biplot.pcaridge, 4, 17, 26	traceplot, 2, 13, 17, 26, 27
biplot.ridge, 26	trans.colors, 29
biplot.ridge(biplot.pcaridge),4	ti alis. co10i 3, 29
	vcov.ridge(ridge), 24
coef.ridge(ridge), 24	vif, <i>31</i>
col2rgb, <i>30</i>	vif.ridge, 3, 30
contour, 7, 8	. 3 - 7 - 7
contour.default,8	wire3d, <i>20</i>