Package 'NormalLaplace'

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Title The Normal Laplace Distribution

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Description Functions for the normal Laplace distribution. Currently, it provides limited functionality. Density, distribution and quantile functions, random number generation, and moments are provided.

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`NormalLaplace-package` *The Package 'NormalLaplace': Summary Information*

Description

This package provides a collection of functions for Normal Laplace distributions. Functions are provided for the density function, distribution function, quantiles and random number generation. The mean, variance, skewness and kurtosis of a given Normal Laplace distribution are given by `nlMean`, `nlVar`, `nlSkew`, and `nlKurt` respectively.

Author(s)

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References

William J. Reed. (2006) The Normal-Laplace Distribution and Its Relatives. In *Advances in Distribution Theory, Order Statistics and Inference*, pp. 61–74. Birkhäuser, Boston.

See Also

`dnl`, `millsR`, `NormalLaplaceMeanVar`

`MillsRatio` *Mills Ratio*

Description

Calculates the Mills ratio

Usage

```
millsR(y, log = FALSE)
```

Arguments

`y`	Numeric. Value at which the Mills' Ratio is evaluated.
`log`	Logical. If `log = TRUE`, Mills' Ratios are given as log(millsR).

Details

The function calculates the Mills' Ratio. Since the Mill's Ratio converges to zero for large positive z and infinity for large negative z. The range over which the logarithm of the Mill's ratio may be calculated is greater than that for which the Mill's ratio itself may be calculated.

Value

The Mills' Ratio is

$$R(z) = \frac{1 - \Phi(z)}{\phi(z)}$$

where $\Phi(z)$ and $\phi(z)$ are respectively the distribution function and density function of the standard normal distribution.

Author(s)

David Scott <d.scott@auckland.ac.nz>, Jason Shicong Fu

Examples

```
## compare millsR calculated directly with the millsR calculated
## by transforming to log scale and then back-transformed
millsR(1:10)
exp(millsR(1:10, log = TRUE))
exp(millsR(10*(1:10)))
exp(millsR(10*(1:10), log = TRUE))
```

---

nlCheckPars *Check Parameters of the Normal Laplace Distribution*

Description

Given a set of parameters for the normal Laplace distribution, the functions checks the validity of each parameter and if they and if they correspond to the boundary cases.

Usage

```
nlCheckPars(param)
```

Arguments

param	Numeric. Parameter values for the normal Laplace distribution.

Details

The vector param takes the form c(mu, sigma, alpha, beta).

If any of sigma, alpha or beta is negative or NA, an error is returned.

Author(s)

David Scott <d.scott@auckland.ac.nz>, Simon Potter

References

William J. Reed. (2006) The Normal-Laplace Distribution and Its Relatives. In *Advances in Distribution Theory, Order Statistics and Inference*, pp. 61–74. Birkhäuser, Boston.

Examples

```
## Correct parameters
nlCheckPars(c(0, 1.5, 1, 2))
nlCheckPars(c(3, 1, 1.5, 2))

## Incorrect parameters, each error providing a different error message
nlCheckPars(c(2, -1, 1, 1))          # invalid sigma
nlCheckPars(c(2, 1, -1, 2))          # invalid alpha
nlCheckPars(c(0, 1, 2, -1))          # invalid beta
nlCheckPars(c(0, -0.01, -0.1, 1))    # sigma and alpha incorrect
nlCheckPars(c(2, -0.5, 1, -0.2))     # sigma and beta incorrect
nlCheckPars(c(1, 1, -0.2, -1))       # alpha and beta incorrect
nlCheckPars(c(0, -0.1, -0.2, -0.3))  # all three parameters erroneous
nlCheckPars(c(0.5, NA, 1, 1))        # NA introduced
nlCheckPars(c(-1, 1, 1))             # incorrect number of parameters
```

`nlFit` *Fit the Normal Laplace Distribution to Data*

Description

Fits a normal Laplace distribution to data. Displays the histogram, log-histogram (both with fitted densities), Q-Q plot and P-P plot for the fit which has the maximum likelihood.

Usage

```
  nlFit(x, freq = NULL, breaks = "FD", paramStart = NULL,
        startMethod = "Nelder-Mead",
        startValues = c("MoM", "US"),
        method = c("Nelder-Mead", "BFGS", "L-BFGS-B",
                   "nlm", "nlminb"),
        hessian = FALSE,
        plots = FALSE, printOut = FALSE,
        controlBFGS = list(maxit = 200),
        controlLBFGSB = list(maxit = 200),
        controlNLMINB = list(),
        controlNM = list(maxit = 1000),
        maxitNLM = 1500, ...)
  ## S3 method for class 'nlFit'
print(x, digits = max(3, getOption("digits") - 3), ...)
  ## S3 method for class 'nlFit'
plot(x, which = 1:4,
    plotTitles = paste(c("Histogram of ","Log-Histogram of ",
```

```
                        "Q-Q Plot of ","P-P Plot of "), x$obsName,
                      sep = ""),
    ask = prod(par("mfcol")) < length(which) & dev.interactive(), ...)
  ## S3 method for class 'nlFit'
coef(object, ...)
  ## S3 method for class 'nlFit'
vcov(object, ...)
```

Arguments

x	Data vector for nlFit.
freq	A vector of weights with length equal to length(x).
breaks	Breaks for plotted histogram, defaults to those generated by hist(x, right = FALSE, plot = FALSE).
paramStart	A user specified starting parameter vector taking the form c(mu, sigma, alpha, beta).
startMethod	Method used by nlFitStart in calls to optim.
startValues	Code giving the method of determining starting values for finding the maximum likelihood estimate of the parameters.
method	Different optimisation methods to consider. See **Details**.
hessian	Logical. If TRUE the value of the Hessian is returned.
plots	Logical. If FALSE the printing of the histogram, log-histogram, Q-Q plot and P-P plot are suppressed.
printOut	Logical. If FALSE the printing of the results of fitting will be suppressed.
controlBFGS	A list of control parameters for optim when using the "BFGS" method of optimisation.
controlLBFGSB	A list of control parameters for optim when using the "L-BFGS-B" method of optimisation.
controlNLMINB	A list of control parameters for optim when using the "nlminb" method of optimisation.
controlNM	A list of control parameters for optim when using the "Nelder-Mead" method of optimisation.
maxitNLM	A positive integer specifying the maximum number of iterations that are to be undertaken when using the "nlm" method of optimisation.
object	Object of class "nlFit" for print.nlFit, plot.nlFit, coef.nlFit and vcov.nlFit.
digits	Desired number of digits to be shown when the object is printed.
which	If a subset of the plots if required, specify a subset of the numbers 1:4.
plotTitles	Titles to appear as the main title above the plots.
ask	Logical. If TRUE, the user is *ask*ed before each plot, see par(ask = .).
...	Passes arguments to par, hist, logHist, qqnl and ppnl. For the print, coef and vcov methods this parameter has no effect.

Details

`startMethod` must be `"Nelder-Mead"`.

`startValues` can only be `"MoM"` when using the Method of Moments for estimation, or `"US"` for user-supplied parameters. For details regarding the use of `paramStart`, `startMethod` and `startValues`, see `nlFitStart`.

Three optimisation methods are available for use:

`"BFGS"` Uses the quasi-Newton method `"BFGS"` as documented in `optim`.

`"L-BFGS-B"` Uses the constrained method `"L-BFGS-B"` as documented in `optim`.

`"Nelder-Mead"` Uses an implementation of the Nelder and Mead method as documented in `optim`.

`"nlm"` Uses the `nlm` function in R.

`"nlminb"` Uses the `nlminb` function in R, with constrained parameters.

For details on how to pass control information for optimisation using `optim` and `nlm`, see `optim` and `nlm`.

When `method = "nlm"` or `method = "nlm"` is used, warnings may be produced. However, these do not appear to be problematic.

Value

A list with components:

`param`	A vector giving the maximum likelihood estimate of parameters, as `c(mu, sigma, alpha, beta)`.
`maxLik`	The value of maximised log-likelihood.
`hessian`	If `hessian` was set to `TRUE`, the value of the Hessian. Not present otherwise.
`method`	Optimisation method used.
`conv`	Convergence code. See the relevant documentation (either `optim` or `nlm`) for details on convergence.
`iter`	Number of iterations made by the optimisation routine.
`obs`	The data used to fit the normal Laplace distribution.
`obsName`	A character vector with the actual `x` argument name.
`paramStart`	Starting value of parameters returned by call to `nlFitStart`.
`svName`	Descriptive name for the method of finding start values.
`startValues`	Acronym for the method of finding start values.
`breaks`	The cell boundaries found by a call to `hist`.
`midpoints`	The cell midpoints found by a call to `hist`.
`empDens`	The estimated density found by a call to `hist`.

Author(s)

David Scott <d.scott@auckland.ac.nz>, Simon Potter

See Also

optim, nlm, par, hist, logHist, qqnl, ppnl, dnl and nlFitStart.

Examples

```
param <- c(0, 2, 1, 1)
dataVector <- rnl(1000, param = param)

## Let's see how well nlFit works
nlFit(dataVector)
nlFit(dataVector, plots = TRUE)
fit <- nlFit(dataVector)
par(mfrow = c(1, 2))
plot(fit, which = c(1, 3))  # See only histogram and Q-Q plot
```

---

nlFitStart *Find Starting Values for Fitting a Normal Laplace Distribution*

Description

Finds starting values for input to a maximum likelihood routine for fitting a normal Laplace distribution to data.

Usage

```
nlFitStart(x, breaks = "FD",
           paramStart = NULL,
           startValues = c("MoM", "US"),
           startMethodMoM = "Nelder-Mead", ...)
nlFitStartMoM(x, startMethodMoM = "Nelder-Mead", ...)
```

Arguments

x	Data vector.
breaks	Breaks for histogram. If missing, defaults to those generated by hist(x, right = FALSE, plot = FALSE).
paramStart	Starting values for parameter vector if startValues = "US".
startValues	Vector of the different starting value methods to consider. See **Details**.
startMethodMoM	Method used by call to optim in finding method of moments estimates.
...	Passes arguments to optim.

Details

Possible values of the argument `startValues` are the following:

`"US"` User-supplied.

`"MoM"` Method of moments.

If `startValues = "US"` then a value must be supplied for `paramStart`.

If `startValues = "MoM"`, `nlFitStartMoM` is called.

If `startValues = "MoM"` an initial optimisation is needed to find the starting values. These optimisations call `optim`.

Value

`nlFitStart` returns a list with components:

`paramStart`	A vector with elements `mu`, `sigma`, `alpha` and `beta` giving the starting value of param.
`xName`	A character string with the actual `x` argument name.
`breaks`	The cell boundaries found by a call to `hist`.
`midpoints`	The cell midpoints found by a call to `hist`.
`empDens`	The estimated density found by a call to `hist`.

`nlFitStartMoM` returns only the method of moments estimates as a vector with elements `mu`, `sigma`, `alpha` and `beta`.

Author(s)

David Scott <d.scott@auckland.ac.nz>, Simon Potter

See Also

`dnl`, `nlFit`, `hist`, and `optim`.

Examples

```
param <- c(2, 2, 1, 1)
dataVector <- rnl(500, param = param)
nlFitStart(dataVector, startValues = "MoM")
```

nlPlots *Normal Laplace Quantile-Quantile and Percent-Percent Plots*

Description

qqnl produces a normal Laplace Q-Q plot of the values in y.

ppnl produces a normal Laplace P-P (percent-percent) or probability plot of the values in y.

Graphical parameters may be given as arguments to qqnl, and ppnl.

Usage

```
qqnl(y, mu = 0, sigma = 1, alpha = 1, beta = 1,
     param = c(mu, sigma, alpha, beta),
     main = "Normal Laplace Q-Q Plot",
     xlab = "Theoretical Quantiles",
     ylab = "Sample Quantiles",
     plot.it = TRUE, line = TRUE, ...)
ppnl(y, mu = 0, sigma = 1, alpha = 1, beta = 1,
     param = c(mu, sigma, alpha, beta),
     main = "Normal Laplace P-P Plot",
     xlab = "Uniform Quantiles",
     ylab = "Probability-integral-transformed Data",
     plot.it = TRUE, line = TRUE, ...)
```

Arguments

y	The data sample.
mu	μ is the location parameter. By default this is set to 0.
sigma	σ is the variance parameter of the distribution. A default value of 1 has been set.
alpha	α is a skewness parameter, with a default value of 1.
beta	β is a shape parameter, by default this is 1.
param	Parameters of the normal Laplace distribution.
xlab, ylab, main	Plot labels.
plot.it	Logical. Should the result be plotted?
line	Add line through origin with unit slope.
...	Further graphical parameters.

Value

For qqnl and ppnl, a list with components:

x	The x coordinates of the points that are to be plotted.
y	The y coordinates of the points that are to be plotted.

References

Wilk, M. B. and Gnanadesikan, R. (1968) Probability plotting methods for the analysis of data. *Biometrika*. **55**, 1–17.

See Also

ppoints, dnl, nlFit

Examples

```
par(mfrow = c(1, 2))
param <- c(2, 2, 1, 1)
y <- rnl(200, param = param)
qqnl(y, param = param, line = FALSE)
abline(0, 1, col = 2)
ppnl(y, param = param)
```

NormalLaplaceDistribution

Normal Laplace Distribution

Description

Density function, distribution function, quantiles and random number generation for the normal Laplace distribution, with parameters μ (location), δ (scale), β (skewness), and ν (shape).

Usage

```
dnl(x, mu = 0, sigma = 1, alpha = 1, beta = 1,
    param = c(mu,sigma,alpha,beta))
pnl(q, mu = 0, sigma = 1, alpha = 1, beta = 1,
    param = c(mu,sigma,alpha,beta))
qnl(p, mu = 0, sigma = 1, alpha = 1, beta = 1,
    param = c(mu,sigma,alpha,beta),
    tol = 10^(-5), nInterpol = 100, subdivisions = 100, ...)
rnl(n, mu = 0, sigma = 1, alpha = 1, beta = 1,
    param = c(mu,sigma,alpha,beta))
```

Arguments

x, q	Vector of quantiles.
p	Vector of probabilities.
n	Number of random variates to be generated.
mu	Location parameter μ, default is 0.
sigma	Scale parameter σ, default is 1.

alpha	Skewness parameter α, default is 1.
beta	Shape parameter β, default is 1.
param	Specifying the parameters as a vector of the form c(mu,sigma,alpha,beta).
tol	Specified level of tolerance when checking if parameter beta is equal to 0.
subdivisions	The maximum number of subdivisions used to integrate the density and determine the accuracy of the distribution function calculation.
nInterpol	Number of points used in qnl for cubic spline interpolation of the distribution function.
...	Passes arguments to uniroot.

Details

Users may either specify the values of the parameters individually or as a vector. If both forms are specified, then the values specified by the vector param will overwrite the other ones.

The density function is

$$f(y) = \frac{\alpha\beta}{\alpha+\beta}\phi\left(\frac{y-\mu}{\sigma}\right)\left[R\left(\alpha\sigma - \frac{(y-\mu)}{\sigma}\right) + R\left(\beta\sigma + \frac{(y-\mu)}{\sigma}\right)\right]$$

The distribution function is

$$F(y) = \Phi\left(\frac{y-\mu}{\sigma}\right) - \phi\left(\frac{y-\mu}{\sigma}\right)\left[\beta R(\alpha\sigma - \frac{y-\mu}{\sigma}) - \alpha R\left(\beta\sigma + \frac{y-\mu}{\sigma}\right)\right]/(\alpha+\beta)$$

The function $R(z)$ is the Mills' Ratio, see millsR.

Generation of random observations from the normal Laplace distribution using rnl is based on the representation

$$Y \sim Z + W$$

where Z and W are independent random variables with

$$Z \sim N(\mu, \sigma^2)$$

and W following an asymmetric Laplace distribution with pdf

$$f_W(w) = \begin{cases} (\alpha\beta)/(\alpha+\beta)e^{\beta w} & \text{for } w \leq 0 \\ (\alpha\beta)/(\alpha+\beta)e^{-\beta w} & \text{for } w > 0 \end{cases}$$

Value

dnl gives the density function, pnl gives the distribution function, qnl gives the quantile function and rnl generates random variates.

Author(s)

David Scott <d.scott@auckland.ac.nz>, Jason Shicong Fu

References

William J. Reed. (2006) The Normal-Laplace Distribution and Its Relatives. In *Advances in Distribution Theory, Order Statistics and Inference*, pp. 61–74. Birkhäuser, Boston.

Examples

```
param <- c(0,1,3,2)
par(mfrow = c(1,2))


## Curves of density and distribution
curve(dnl(x, param = param), -5, 5, n = 1000)
title("Density of the Normal Laplace Distribution")
curve(pnl(x, param = param), -5, 5, n = 1000)
title("Distribution Function of the Normal Laplace Distribution")


## Example of density and random numbers
 par(mfrow = c(1,1))
 param1  <-  c(0,1,1,1)
 data1  <-  rnl(1000, param = param1)
 curve(dnl(x, param = param1),
       from = -5, to = 5, n = 1000, col = 2)
 hist(data1, freq = FALSE, add = TRUE)
 title("Density and Histogram")
```

NormalLaplaceMeanVar *Mean, Variance, Skewness and Kurtosis of the Normal Laplace Distribution.*

Description

Functions to calculate the mean, variance, skewness and kurtosis of a specified normal Laplace distribution.

Usage

```
nlMean(mu = 0, sigma = 1, alpha = 1, beta = 1,
         param = c(mu, sigma, alpha, beta))
nlVar(mu = 0, sigma = 1, alpha = 1, beta = 1,
         param = c(mu, sigma, alpha, beta))
nlSkew(mu = 0, sigma = 1, alpha = 1, beta = 1,
         param = c(mu, sigma, alpha, beta))
nlKurt(mu = 0, sigma = 1, alpha = 1, beta = 1,
         param = c(mu, sigma, alpha, beta))
```

Arguments

mu	Location parameter μ, default is 0.
sigma	Scale parameter σ, default is 1.
alpha	Skewness parameter α, default is 1.
beta	Shape parameter β, default is 1.
param	Specifying the parameters as a vector of the form c(mu, sigma, alpha, beta).

Details

Users may either specify the values of the parameters individually or as a vector. If both forms are specified, then the values specified by the vector param will overwrite the other ones.

The mean function is

$$E(Y) = \mu + 1/\alpha - 1/\beta.$$

The variance function is

$$V(Y) = \sigma^2 + 1/\alpha^2 + 1/\beta^2.$$

The skewness function is

$$\Upsilon = [2/\alpha^3 - 2/\beta^3]/[\sigma^2 + 1/\alpha^2 + 1/\beta^2]^{3/2}.$$

The kurtosis function is

$$\Gamma = [6/\alpha^4 + 6/\beta^4]/[\sigma^2 + 1/\alpha^2 + 1/\beta^2]^2.$$

Value

nlMean gives the mean of the skew hyperbolic nlVar the variance, nlSkew the skewness, and nlKurt the kurtosis.

Author(s)

David Scott <d.scott@auckland.ac.nz>, Jason Shicong Fu

References

William J. Reed. (2006) The Normal-Laplace Distribution and Its Relatives. In *Advances in Distribution Theory, Order Statistics and Inference*, pp. 61–74. Birkhäuser, Boston.

Examples

```
param <- c(10,1,5,9)
nlMean(param = param)
nlVar(param = param)
nlSkew(param = param)
nlKurt(param = param)


curve(dnl(x, param = param), -10, 10)
```

summary.nlFit *Summarizing Normal Laplace Distribution Fit*

Description

summary Method for class "nlFit".

Usage

```
  ## S3 method for class 'nlFit'
summary(object, ...)
  ## S3 method for class 'summary.nlFit'
print(x,
        digits = max(3, getOption("digits") - 3), ...)
```

Arguments

object	An object of class "nlFit", resulting from a call to nlFit.
x	An object of class "summary.nlFit" resulting from a call to summary.nlFit.
digits	The number of significant digits to use when printing.
...	Further arguments passed to or from other methods.

Details

summary.nlFit calculates standard errors for the estimates of μ, σ, α, and β of the normal laplace distribution parameter vector param if the Hessian from the call to nlFit is available.

Value

If the Hessian is available, summary.nlFit computes standard errors for the estimates of μ, σ, α, and β, and adds them to object as object$sds. Otherwise, no calculations are performed and the composition of object is unaltered.

summary.nlFit invisibly returns object with class changed to summary.nlFit.

See nlFit for the composition of an object of class nlFit.

print.summary.nlFit prints a summary in the same format as print.nlFit when the Hessian is not available from the fit. When the Hessian is available, the standard errors for the parameter estimates are printed in parentheses beneath the parameter estimates, in the manner of fitdistr in the package MASS.

See Also

nlFit, summary.

Examples

```
## Continuing the  nlFit() example:
param <- c(2, 2, 1, 1)
dataVector <- rnl(500, param = param)
fit <- nlFit(dataVector, hessian = TRUE)
print(fit)
summary(fit)
```

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