

Package ‘MonoPoly’

April 5, 2016

Type Package

Title Functions to Fit Monotone Polynomials

Version 0.3-8

Date 2016-04-04

Description Functions for fitting monotone polynomials to data.

License GPL (>=2)

Depends R (>= 3.1.0), quadprog

LazyData yes

Encoding UTF-8

R topics documented:

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| | |
|-------------|-----------------------------------|
| coef.monpol | <i>Extract Model Coefficients</i> |
|-------------|-----------------------------------|

Description

coef method for ‘monpol’ objects.

Usage

```
## S3 method for class 'monpol'
coef(object, scale = c("original", "fitted"), type = c("beta", "monpar"), ...)
```

Arguments

| | |
|--------|---|
| object | A ‘monpol’ object. |
| scale | Extract coefficients on the original scale of the data or on the scale used during fitting. |
| type | Extract coefficients in the ‘beta’ parameterisation of the polynomial or for the monotone parameterisation used in the algorithm. |
| ... | Additional optional arguments. At present no optional arguments are used. |

Details

This is the `coef` method for objects inheriting from class `"monpol"`.

Value

Coefficients extracted from the model object `object`.

Author(s)

Berwin A Turlach

| | |
|---------|--|
| curvPol | <i>Evaluating the Curvature of Polynomials</i> |
|---------|--|

Description

Function to evaluate the curvature of polynomials

Usage

```
curvPol(x, beta)
```

Arguments

| | |
|------|--|
| x | numerical values at which to evaluate the curvature of polynomials, can be provided in a vector, matrix, array or data frame |
| beta | numerical vector containing the coefficient of the polynomial |

Value

The result of evaluating the curvature of the polynomial at the values in x, returned in the same dimension as x has.

Author(s)

Berwin A Turlach

Examples

```
beta <- c(1,2,1)

x <- 0:10
curvPol(x, beta)
str(curvPol(x, beta))

x <- cbind(0:10, 10:0)
curvPol(x, beta)
str(curvPol(x, beta))

x <- data.frame(x=0:10, y=10:0)
curvPol(x, beta)
str(curvPol(x, beta))
```

evalPol

Evaluating Polynomials

Description

Function to evaluate polynomials in a numerical robust way using the Horner scheme

Usage

```
evalPol(x, beta)
```

Arguments

| | |
|------|---|
| x | numerical values at which to evaluate polynomials, can be provided in a vector, matrix, array or data frame |
| beta | numerical vector containing the coefficient of the polynomial |

Value

The result of evaluating the polynomial at the values in x, returned in the same dimension as x has.

Author(s)

Berwin A Turlach

Examples

```

beta <- c(1,2,1)

x <- 0:10
evalPol(x, beta)
str(evalPol(x, beta))

x <- cbind(0:10, 10:0)
evalPol(x, beta)
str(evalPol(x, beta))

x <- data.frame(x=0:10, y=10:0)
evalPol(x, beta)
str(evalPol(x, beta))

```

fitted.monpol

*Extract Model Fitted Values***Description**

fitted method for ‘monpol’ objects.

Usage

```

## S3 method for class 'monpol'
fitted(object, scale = c("original", "fitted"), ...)

```

Arguments

| | |
|--------|--|
| object | A ‘monpol’ object. |
| scale | Extract fitted values on the original scale of the data or on the scale used during fitting. |
| ... | Additional optionals arguments. At present no optional arguments are used. |

Details

This is the [fitted](#) method for objects inheriting from class "monpol".

Value

Fitted values extracted from the model object object.

Author(s)

Berwin A Turlach

| | |
|---------|----------------|
| hawkins | <i>hawkins</i> |
|---------|----------------|

Description

This data gives x and y variables for the data published in Hawkins' 1994 article. This data was originally simulated from a standard cubic polynomial with equally spaced x values between -1 and 1.

Format

A data frame with 50 simulated observations on the following 2 variables.

y a numeric vector

x a numeric vector

References

Hawkins, D. M. (1994) Fitting monotonic polynomials to data. *Computational Statistics* **9**(3): 233–247.

Examples

```
data(hawkins)
```

| | |
|------------|---|
| ismonotone | <i>Check whether a polynomial is monotone</i> |
|------------|---|

Description

Function to check whether a polynomial is monotone over a given interval.

Usage

```
ismonotone(object, ...)

## S3 method for class 'monpol'
ismonotone(object, a = -Inf, b = Inf, EPS = 1e-06, ...)

## Default S3 method:
ismonotone(object, a = -Inf, b = Inf, EPS = 1e-06, ...)
```

Arguments

| | |
|--------|--|
| object | Either an object of class ' monpol ' or a numeric vector containing the coefficient of the polynomial. |
| a | Lower limit of the interval over which the polynomial should be monotone. |

| | |
|-----|---|
| b | Upper limit of the interval over which the polynomial should be montone. |
| EPS | Numerical precision, values with absolute value smaller than EPS are treated as zero. |
| ... | Further arguments passed to or from other methods. |

Value

TRUE or FALSE depending on whether the polynomial is montone over (a,b) or not.

Note that due to numerical precision issues it is possible that a polynomial that should be monotone is declared to be not monotone.

Author(s)

Kevin Murray and Berwin A Turlach

Examples

```
fit <- monpol(y~x, w0)
ismonotone(fit)

beta <- c(1,0,2) ## the polynomial 1 + 2*x^2
ismonotone(beta)
ismonotone(beta, a=0)
ismonotone(beta, b=0)
```

| | |
|---------------------|----------------------------------|
| model.matrix.monpol | <i>Construct Design Matrices</i> |
|---------------------|----------------------------------|

Description

model.matrix creates a design (or model) matrix for 'monpol' objects.

Usage

```
## S3 method for class 'monpol'
model.matrix(object, scale = c("original", "fitted"), ...)
```

Arguments

| | |
|--------|---|
| object | A 'monpol' object. |
| scale | Create design matrix on the original scale of the data or on the scale used during fitting. |
| ... | Additional optionals arguments. At present no optional arguments are used. |

Details

This is the `model.matrix` method for objects inheriting from class "monpol".

Value

Design matrix created from the model object object.

Author(s)

Berwin A Turlach

monpol

*Monotone Polynomials***Description**

Determine the least-squares estimates of the parameters of a monotone polynomial

Usage

```
monpol(formula, data, subset, weights, na.action,
       degree = 3, K, start,
       a = -Inf, b=Inf,
       trace = FALSE, plot.it = FALSE,
       control = monpol.control(),
       algorithm = c("Full", "Hawkins", "BCD", "CD1", "CD2"),
       ptype = c("SOS", "Elphinstone", "EHH", "Penttila"),
       ctype = c("cge0", "c2"),
       monotone,
       model=FALSE, x=FALSE, y=FALSE)
```

Arguments

| | |
|-----------|---|
| formula | an object of class " formula " (or one that can be coerced to that class): a symbolic description of the model to be fitted. |
| data | an optional data frame, list or environment (or object coercible by as.data.frame to a data frame) containing the variables in the model. If not found in data, the variables are taken from environment(<code>formula</code>), typically the environment from which <code>monpol</code> is called. |
| subset | an optional vector specifying a subset of observations to be used in the fitting process. |
| weights | an optional vector of weights to be used in the fitting process. Should be <code>NULL</code> or a numeric vector. |
| na.action | a function which indicates what should happen when the data contain NAs. The default is set by the <code>na.action</code> setting of options , and is na.fail if that is unset. The ‘factory-fresh’ default is na.omit . Another possible value is <code>NULL</code> , no action. Value na.exclude can be useful. |
| degree | positive integer, a polynomial with highest power equal to <code>degree</code> will be fitted to the data. |
| K | non-negative integer, a polynomial with highest power $2K + 1$ will be fitted to the data. |
| start | optional starting value for the iterative fitting. |
| a,b | polynomial should be monotone on the interval from <code>a</code> to <code>b</code> . If either parameter is finite, parameterisation “SOS” has to be used. |
| trace | print out information about the progress of the iterative fitting at the start and then every <code>trace</code> iterations. |

| | |
|--------------------------|---|
| <code>plot.it</code> | plot the data and initial fit, then plot current fit every <code>plot.it</code> iterations. |
| <code>control</code> | settings that control the iterative fit; see <code>monpol.control</code> for details. |
| <code>algorithm</code> | algorithm to be used. It is recommended to use either “Full” or “Hawkins”; see both papers in ‘References’ for details. |
| <code>ptype</code> | parameterisation to be used. It is recommended to use the “SOS” parameterisation; see the 2016 paper in ‘References’ for details. |
| <code>ctype</code> | parameterisation to be used; see paper in ‘References’ for details. |
| <code>monotone</code> | only used for parameterisation “SOS” to enforce the kind of monotonicity desired over the interval $[a, b]$, should be “increasing” or “decreasing”. |
| <code>model, x, y</code> | logicals. If TRUE the corresponding components of the fit (the model frame, the model matrix, the response, the QR decomposition) are returned. |

Details

A `monpol` object is a type of fitted model object. It has methods for the generic function `coef`, `fitted`, `formula`, `logLik`, `model.matrix`, `predict`, `print`, `residuals`.

The parameterisation type “SOS” with the “Full” algorithm is currently the recommended fitting procedure and is discussed in the 2016 paper in ‘References’. For this parameterisation the argument `ctype` is ignored.

The “Hawkins” algorithm is also recommended and discussed in both papers in the ‘References’.

The parameterisations “Elphinstone”, “EHH” and “Pentilla”, for which the argument “`ctype`” defines a further variation of parameterisation, work together with algorithms “Full”, “BCD”, “CD1” and “CD2”. These parameterisations and algorithms are discussed in the 2013 paper in ‘References’.

Value

`monpol` returns an object of class “`monpol`”

Author(s)

Berwin A Turlach

References

- Murray, K., Müller, S. and Turlach, B.A. (2016). Fast and flexible methods for monotone polynomial fitting, *Journal of Statistical Computation and Simulation*. Accepted for publication, doi:10.1080/00949655.2016.1191111.
- Murray, K., Müller, S. and Turlach, B.A. (2013). Revisiting fitting monotone polynomials to data, *Computational Statistics* **28**(5): 1989–2005. Doi:10.1007/s00180-012-0390-5.

Examples

```
monpol(y~x, w0)
```

| | |
|----------------|---|
| monpol.control | <i>Control the Iterations in monpol</i> |
|----------------|---|

Description

Allow the user to set some characteristics of the monpol monotone polynomial fitting algorithm.

Usage

```
monpol.control(maxiter = 1000, tol = 1e-05,
               tol1=1e-10, tol2=1e-07, tolqr=1e-07)
```

Arguments

| | |
|---------|---|
| maxiter | A positive integer specifying the maximum number of iterations allowed, used in all algorithms. |
| tol | A positive numeric value specifying an absolute tolerance for determining whether entries in the gradient are zero for algorithms ‘Full’, ‘BCD’, ‘CD1’ and ‘CD2’. |
| tol1 | A positive numeric value, used in algorithm ‘Hawkins’. Any number not smaller than -tol1 is deemed to be non-negative. |
| tol2 | A positive numeric value, used in algorithm ‘Hawkins’. Any number whose absolute value is smaller than tol2 is taken to be zero. |
| tolqr | A positive numeric value, used in algorithm ‘Hawkins’ as tolerance for the QR factorisation of the design matrix. |

Value

A list with exactly five components:

```
maxiter
tol
tol1
tol2
tolqr
```

with meanings as explained under ‘Arguments’.

Author(s)

Berwin A Turlach

See Also

[monpol](#), [monpol.fit](#), [qr](#)

Examples

```
monpol.control(maxiter = 2000)
monpol.control(tolqr = 1e-10)
```

monpol.fit

*Monotone Polynomials***Description**

This is the basic computing engine called by [monpol](#) used to fit monotonic polynomials. These should usually *not* be used directly unless by experienced users.

Usage

```
monpol.fit(x, y, w, K=1, start, trace = FALSE, plot.it = FALSE,
           control = monpol.control(),
           algorithm = c("Full", "Hawkins", "BCD", "CD1", "CD2"),
           ptype = c("Elphinstone", "EHH", "Penttila"),
           ctype = c("cge0", "c2"))
SOSpol.fit(x, y, w = NULL, deg.is.odd, K, start, a, b,
            monotone = c("increasing", "decreasing"),
            trace = FALSE, plot.it = FALSE, type,
            control = monpol.control())
```

Arguments

| | |
|---------------|--|
| x | vector containing the observed values for the regressor variable. |
| y | vector containing the observed values for the response variable; should be of same length as x. |
| w | optional vector of weights; should be of the same length as x if specified. |
| deg.is.odd, K | “deg.is.odd” is a logical, “K” is a non negative integer. If “deg.is.odd” is TRUE then a polynomial with highest power $2K + 1$ will be fitted to the data, otherwise the highest order will be $2K$. |
| start | optional starting value for the iterative fitting. |
| a,b, type | polynomial should be monotone on the interval from <i>a</i> to <i>b</i> ; “type” should be 0 if neither of the boundaries is finite, 1 if <i>a</i> is finite but not <i>b</i> and 2 if both boundaries are finite. |
| monotone | force the desired monotonicity in case the default choice is wrong. |
| trace | print out information about the progress of the iterative fitting at the start and then every trace iterations. |
| plot.it | plot the data and initial fit, then plot current fit every plot.it iterations. |
| control | settings that control the iterative fit; see monpol.control for details. |
| algorithm | algorithm to be used; see monpol for details. |
| ptype | parameterisation to be used; see monpol for details. |
| ctype | parameterisation to be used; see monpol for details. |

Value

a list with components

| | |
|---------------|--|
| par | the fitted parameters. |
| grad | the gradient of the objective function at the fitted parameters. |
| beta | the coefficients of the fitted polynomial in the ‘beta’ parameterisation; on the fitted scale. |
| RSS | the value of the objective function; on the fitted scale. |
| niter | number of iterations. |
| converged | indicates whether algorithm has converged. |
| ptype | input parameter ptype. |
| cotype | input parameter cotype. |
| beta.raw | the coefficients of the fitted polynomial in the ‘beta’ parameterisation; on the original scale. |
| fitted.values | the fitted values; on the fitted scale. |
| residuals | the residuals; on the fitted scale. |
| K | input parameter K. |
| minx | the minimum value in the vector x. |
| sclx | the difference between the maximum and minimum values in the vector x. |
| miny | the minimum value in the vector y. |
| sclx | the difference between the maximum and minimum values in the vector y. |
| algorithm | input parameter algorithm. |

Author(s)

Berwin A Turlach

References

- Murray, K., Müller, S. and Turlach, B.A. (2016). Fast and flexible methods for monotone polynomial fitting, *Journal of Statistical Computation and Simulation*. Accepted for publication, doi:10.1080/00949655.2016.1111111.
- Murray, K., Müller, S. and Turlach, B.A. (2013). Revisiting fitting monotone polynomials to data, *Computational Statistics* **28**(5): 1989–2005. Doi:10.1007/s00180-012-0390-5.

See Also

[monpol](#) which you should use for fitting monotonic polynomials unless you know better.

| | |
|----------------|---|
| predict.monpol | <i>Predicting from Monotone Polynomial Fits</i> |
|----------------|---|

Description

predict.monpol produces predicted values, obtained by evaluating the monotone polynomial in the frame newdata.

Usage

```
## S3 method for class 'monpol'
predict(object, newdata, scale = c("original", "fitted"), ...)
```

Arguments

| | |
|---------|--|
| object | A ‘monpol’ object. |
| newdata | A named list or data frame in which to look for variables with which to predict. If newdata is missing the fitted values at the original data points are returned. |
| scale | Predict values on the original scale of the data or on the scale used during fitting. Data in newdata is assumed to be on the indicated scale. |
| ... | Additional optional arguments. At present no optional arguments are used. |

Details

This is the [predict](#) method for objects inheriting from class "monpol".

Value

predict.monpol produces a vector of predictions.

Author(s)

Berwin A Turlach

| | |
|--------------|--------------------------------------|
| print.monpol | <i>Printing Monotone Polynomials</i> |
|--------------|--------------------------------------|

Description

print method for ‘monpol’ objects.

Usage

```
## S3 method for class 'monpol'
print(x, digits = max(3, getOption("digits") - 3), ...)
```

Arguments

| | |
|--------|---|
| x | A ‘monpol’ object. |
| digits | minimal number of <i>significant</i> digits, see print.default . |
| ... | Additional optional arguments. At present only those additional arguments for coef.monpol are used. |

Details

This is the [print](#) method for objects inheriting from class "monpol".

Value

x returned invisibly.

Author(s)

Berwin A Turlach

| | |
|------------------|--------------------------------|
| residuals.monpol | <i>Extract Model Residuals</i> |
|------------------|--------------------------------|

Description

residuals method for ‘monpol’ objects.

Usage

```
## S3 method for class 'monpol'
residuals(object, scale = c("original", "fitted"), ...)
```

Arguments

| | |
|--------|--|
| object | A ‘monpol’ object. |
| scale | Extract residuals on the original scale of the data or on the scale used during fitting. |
| ... | Additional optional arguments. At present no optional arguments are used. |

Details

This is the [residuals](#) method for objects inheriting from class "monpol".

Value

Residuals extracted from the model object object.

Author(s)

Berwin A Turlach

w0

*Simulated w0 data used in Murray et al. (2013)***Description**

This data set gives simulated data from the function

$$y = 0.1x^3 + e$$

for $e \sim N(0, 0.01^2)$ and x evenly spaced between -1 and 1.

Format

A data frame with 21 observations on the following 2 variables.

y a numeric vector

x a numeric vector

Source

Murray, K., Müller, S. and Turlach, B.A. (2013). Revisiting fitting monotone polynomials to data, *Computational Statistics* **28**(5): 1989–2005. Doi:10.1007/s00180-012-0390-5.

Examples

```
str(w0)
plot(y~x, w0)
monpol(y~x, w0)
```

w2

*Simulated w2 data used in Murray et al. (2013)***Description**

Simulated data from the function

$$y_{ij} = 4\pi - x_i + \cos(x_i - \frac{\pi}{2}) + e_{ij}$$

for $x_i = 0, 1, \dots, 12$; $n_i = 5$ for $i = 0$ and $n_i = 3$ otherwise; $e_{ij} \sim N(0, 0.5^2)$

Format

A data frame with 41 observations on the following 2 variables.

y a numeric vector

x a numeric vector

Source

Murray, K., Müller, S. and Turlach, B.A. (2013). Revisiting fitting monotone polynomials to data, *Computational Statistics* **28**(5): 1989–2005. Doi:10.1007/s00180-012-0390-5.

Examples

```
str(w2)
plot(y~x, w2)
monpol(y~x, w2)
monpol(y~x, w2, K=2)
```

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