

# 1 twosls: Two Stage Least Squares

`twosls` provides consistent estimates for linear regression models with some explanatory variable (the instrumental variable) correlated with the error term. In this situation, ordinary least squares fails to provide consistent estimates. The name two-stage least squares stems from the two regressions in the estimation procedure. In stage one, an ordinary least squares prediction of the instrumental variable is obtained from regressing it on the instrument variables. In stage two, the coefficients of interest are estimated using ordinary least square after substituting the instrumental variable by its predictions from stage one.

## 1.0.1 Syntax

```
> fml <- list ("mu" = Y ~ X + Z,
              "inst" = Z ~ W + X)
> z.out <- zelig(formula = fml, model = "twosls", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out)
```

## 1.0.2 Inputs

`twosls` regression take the following inputs:

- **formula**: a list of the main equation and instrumental variable equation. The first object in the list `mu` corresponds to the regression model needs to be estimated. The second list object `inst` specifies the regression model for the instrumental variable Z. For example:

```
> fml <- list ("mu" = Y ~ X + Z,
+             "inst" = Z ~ W + X)
```

- Y: the dependent variable of interest.
- Z: the instrumental variable.
- W: exogenous instrument variables.

## 1.0.3 Additional Inputs

`twosls` takes the following additional inputs for model specifications:

- **TX**: an optional matrix to transform the regressor matrix and, hence, also the coefficient vector (see 1.0.4). Default is `NULL`.
- **rcovformula**: formula to calculate the estimated residual covariance matrix (see 1.0.4). Default is equal to 1.
- **probdfsys**: use the degrees of freedom of the whole system (in place of the degrees of freedom of the single equation to calculate probability values for the t-test of individual parameters.

- **single.eq.sigma**: use different  $\sigma^2$  for each single equation to calculate the covariance matrix and the standard errors of the coefficients.
- **solveto1**: tolerance level for detecting linear dependencies when inverting a matrix or calculating a determinant. Default is **solveto1**=Machine\$double.eps.
- **saveMemory**: logical. Save memory by omitting some calculation that are not crucial for the basic estimate (e.g McElroy's  $R^2$ ).

#### 1.0.4 Details

- **TX**: The matrix **TX** transforms the regressor matrix ( $X$ ) by  $X* = X \times TX$ . Thus, the vector of coefficients is now  $b = TX \times b*$  where  $b$  is the original(stacked) vector of all coefficients and  $b*$  is the new coefficient vector that is estimated instead. Thus, the elements of vector  $b$  and  $b_i = \sum_j TX_{ij} \times b_j*$ . The  $TX$  matrix can be used to change the order of the coefficients and also to restrict coefficients (if  $TX$  has less columns than it has rows).
- **rcovformula**: The formula to calculate the estimated covariance matrix of the residuals( $\hat{\Sigma}$ ) can be one of the following (see Judge et al., 1955, p.469): if **rcovformula**= 0:

$$\hat{\sigma}_{ij} = \frac{\hat{e}_i' \hat{e}_j}{T}$$

if **rcovformula**= 1 or **rcovformula**='geomean':

$$\hat{\sigma}_{ij} = \frac{\hat{e}_i' \hat{e}_j}{\sqrt{(T - k_i) \times (T - k_j)}}$$

if **rcovformula**= 2 or **rcovformula**='Theil':

$$\hat{\sigma}_{ij} = \frac{\hat{e}_i' \hat{e}_j}{T - k_i - k_j + \text{tr}[X_i(X_i'X_i)^{-1}X_i'X_j(X_j'X_j)^{-1}X_j']}$$

if **rcovformula**= 3 or **rcovformula**='max':

$$\hat{\sigma}_{ij} = \frac{\hat{e}_i' \hat{e}_j}{T - \max(k_i, k_j)}$$

If  $i = j$ , formula 1, 2, and 3 are equal. All these three formulas yield unbiased estimators for the diagonal elements of the residual covariance matrix. If *ineqj*, only formula 2 yields an unbiased estimator for the residual covariance matrix, but it is not necessarily positive semidefinite. Thus, it is doubtful whether formula 2 is really superior to formula 1

### 1.0.5 Examples

Attaching the example dataset:

```
> data(klein)
```

Formula:

```
> formula <- list(mu1=C~Wtot + P + P1,  
+                 mu2=I~P + P1 + K1,  
+                 mu3=Wp~ X + X1 + Tm,  
+                 inst= ~ P1 + K1 + X1 + Tm + Wg + G)
```

Estimating the model using twosls:

```
> z.out<-zelig(formula=formula, model="twosls",data=klein)
```

How to cite this model in Zelig:

Ferdinand Alimadhi, Ying Lu, and Elena Villalon. 2013.

"twosls"

in Kosuke Imai, Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software,"

<http://gking.harvard.edu/zelig>

```
> summary(z.out)
```

systemfit results

method: 2SLS

	N	DF	SSR	detRCov	OLS-R2	McElroy-R2
system	63	51	60.531	0.495617	0.969562	0.993214

	N	DF	SSR	MSE	RMSE	R2	Adj R2
mu1	21	17	19.7649	1.162638	1.078257	0.979005	0.975301
mu2	21	17	30.6494	1.802905	1.342723	0.878533	0.857098
mu3	21	17	10.1167	0.595103	0.771429	0.987273	0.985027

The covariance matrix of the residuals

	mu1	mu2	mu3
mu1	1.162638	0.451038	-0.468245
mu2	0.451038	1.802905	0.303630
mu3	-0.468245	0.303630	0.595103

The correlations of the residuals

	mu1	mu2	mu3
mu1	1.000000	0.311533	-0.562930
mu2	0.311533	1.000000	0.293131
mu3	-0.562930	0.293131	1.000000

2SLS estimates for 'mu1' (equation 1)

Model Formula:  $C \sim W_{tot} + P + P1$

Instruments:  $\sim P1 + K1 + X1 + Tm + Wg + G$

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	16.3990728	1.4088073	11.64039	1.6004e-09 ***
Wtot	0.8082108	0.0425534	18.99286	6.9500e-13 ***
P	0.0720813	0.1439879	0.50061	0.62307
P1	0.1742361	0.1260083	1.38274	0.18464

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.078257 on 17 degrees of freedom

Number of observations: 21 Degrees of Freedom: 17

SSR: 19.764853 MSE: 1.162638 Root MSE: 1.078257

Multiple R-Squared: 0.979005 Adjusted R-Squared: 0.975301

2SLS estimates for 'mu2' (equation 2)

Model Formula:  $I \sim P + P1 + K1$

Instruments:  $\sim P1 + K1 + X1 + Tm + Wg + G$

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	20.9498400	10.3377708	2.02653	0.0586938 .
P	0.1284295	0.2712099	0.47354	0.6418500
P1	0.6346591	0.2448306	2.59224	0.0189823 *
K1	-0.1608303	0.0487085	-3.30190	0.0042128 **

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.342723 on 17 degrees of freedom

Number of observations: 21 Degrees of Freedom: 17

SSR: 30.649387 MSE: 1.802905 Root MSE: 1.342723

Multiple R-Squared: 0.878533 Adjusted R-Squared: 0.857098

2SLS estimates for 'mu3' (equation 3)

Model Formula:  $Wp \sim X + X1 + Tm$

Instruments:  $\sim P1 + K1 + X1 + Tm + Wg + G$

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	1.5714723	1.2831206	1.22473	0.23737862
X	0.4253396	0.0402022	10.58000	6.7336e-09 ***
X1	0.1594488	0.0437152	3.64744	0.00199274 **
Tm	0.1336876	0.0325964	4.10130	0.00074462 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.771429 on 17 degrees of freedom  
Number of observations: 21 Degrees of Freedom: 17  
SSR: 10.116746 MSE: 0.595103 Root MSE: 0.771429  
Multiple R-Squared: 0.987273 Adjusted R-Squared: 0.985027

Set explanatory variables to their default (mean/mode) values

```
> x.out <- setx(z.out)
```

Simulate draws from the posterior distribution:

```
> s.out <- sim(z.out, x=x.out)
> summary(s.out)
```

Model: twosls

Number of simulations: 1000

Values of X

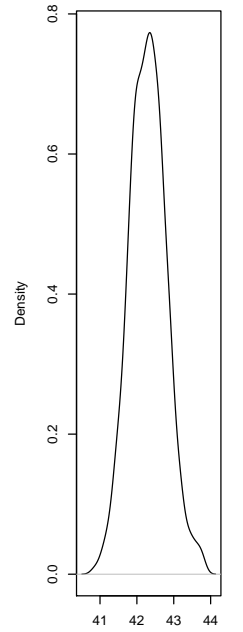
	(Intercept)	Wtot	P	P1	K1	X	X1	Tm
[1,]	1	28.2	12.4	12.7	182.8	60.05714	57.98571	0

Expected Value: E(Y|X)

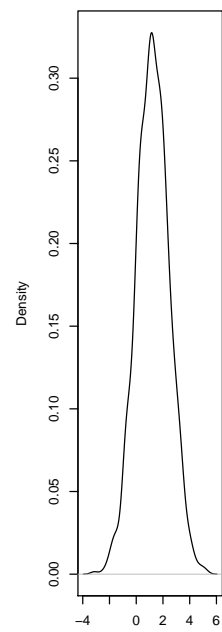
	mean	sd	50%	2.5%	97.5%
mu1	42.297	0.496	42.299	41.372	43.339
mu2	1.230	1.213	1.215	-1.008	3.537
mu3	36.356	0.168	36.359	36.025	36.661

Plot the quantities of interest

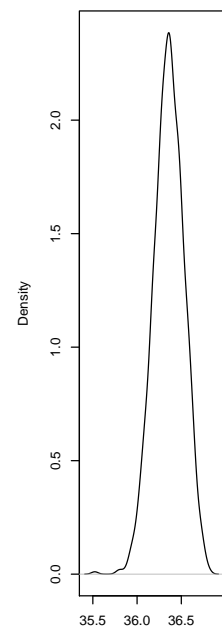
mu1: Expected Value:  $E(Y|X)$     mu2: Expected Value:  $E(Y|X)$     mu3: Expected Value:  $E(Y|X)$



N = 1000 Bandwidth = 0.1122



N = 1000 Bandwidth = 0.2742



N = 1000 Bandwidth = 0.0379

### 1.0.6 Model

Let's consider the following regression model,

$$Y_i = X_i\beta + Z_i\gamma + \epsilon_i, \quad i = 1, \dots, N$$

where  $Y_i$  is the dependent variable,  $X_i = (X_{1i}, \dots, X_{Ni})$  is the vector of explanatory variables,  $\beta$  is the vector of coefficients of the explanatory variables  $X_i$ ,  $Z_i$  is the problematic explanatory variable, and  $\gamma$  is the coefficient of  $Z_i$ . In the equation, there is a direct dependence of  $Z_i$  on the structural disturbances of  $\epsilon$ .

- The *stochastic component* is given by

$$\epsilon_i \sim \mathcal{N}(0, \sigma^2), \quad \text{and} \quad \text{cov}(Z_i, \epsilon_i) \neq 0,$$

- The *systematic component* is given by:

$$\mu_i = E(Y_i) = X_i\beta + Z_i\gamma,$$

To correct the problem caused by the correlation of  $Z_i$  and  $\epsilon$ , two stage least squares utilizes two steps:

- *Stage 1:* A new instrumental variable  $\hat{Z}$  is created for  $Z_i$  which is the ordinary least squares predictions from regressing  $Z_i$  on a set of exogenous instruments  $W$  and  $X$ .

$$\widehat{Z}_i = \widetilde{W}_i[(\widetilde{W}^\top \widetilde{W})^{-1} \widetilde{W}^\top Z]$$

where  $\widetilde{W} = (W, X)$

- *Stage 2:* Substitute for  $\hat{Z}_i$  for  $Z_i$  in the original equation, estimate  $\beta$  and  $\gamma$  by ordinary least squares regression of  $Y$  on  $X$  and  $\hat{Z}$  as in the following equation.

$$Y_i = X_i\beta + \widehat{Z}_i\gamma + \epsilon_i, \quad \text{for } i = 1, \dots, N$$

### 1.0.7 See Also

For information about three stage least square regression, see Section ?? and `help(3sls)`. For information about seemingly unrelated regression, see Section ?? and `help(sur)`.

### 1.0.8 Quantities of Interest

### 1.0.9 Output Values

The output of each Zelig command contains useful information which you may view. For example, if you run:

```
z.out <- zelig(formula=fml, model = "twosls", data)
```

then you may examine the available information in `z.out` by using `names(z.out)`, see the draws from the posterior distribution of the `coefficients` by using `z.out$coefficients`, and view a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below:

- `h`: matrix of all (diagonally stacked) instrumental variables.
- `single.eq.sigma`: different  $\sigma^2$ s for each single equation?.
- `zelig.data`: the input data frame if `save.data = TRUE`.
- `method`: Estimation method.
- `g`: number of equations.
- `n`: total number of observations.
- `k`: total number of coefficients.
- `ki`: total number of linear independent coefficients.
- `df`: degrees of freedom of the whole system.
- `iter`: number of iteration steps.
- `b`: vector of all estimated coefficients.
- `t`:  $t$  values for  $b$ .
- `se`: estimated standard errors of  $b$ .
- `bt`: coefficient vector transformed by  $TX$ .
- `p`:  $p$  values for  $b$ .
- `bcov`: estimated covariance matrix of  $b$ .
- `btcov`: covariance matrix of  $bt$ .
- `rcov`: estimated residual covariance matrix.
- `drcov`: determinant of `rcov`.
- `rcor`: estimated residual correlation matrix.
- `olsr2`: system OLS R-squared value.
- `y`: vector of all (stacked) endogenous variables.
- `x`: matrix of all (diagonally stacked) regressors.



- **data**: data frame of the whole system (including instruments).
- **TX**: matrix used to transform the regressor matrix.
- **rcovformula**: formula to calculate the estimated residual covariance matrix.
- **probdfsys**: system degrees of freedom to calculate probability values?.
- **solveto1**: tolerance level when inverting a matrix or calculating a determinant.
- **eq**: a list that contains the results that belong to the individual equations.
- **eqnlabel\***: the equation label of the *ith* equation (from the labels list).
- **formula\***: model formula of the *ith* equation.
- **n\***: number of observations of the *ith* equation.
- **k\***: number of coefficients/regressors in the *ith* equation (including the constant).
- **ki\***: number of linear independent coefficients in the *ith* equation (including the constant differs from *k* only if there are restrictions that are not cross equation).
- **df\***: degrees of freedom of the *ith* equation.
- **b\***: estimated coefficients of the *ith* equation.
- **se\***: estimated standard errors of *b* of the *ith* equation.
- **t\***: *t* values for *b* of the *ith* equation.
- **p\***: *p* values for *b* of the *ith* equation.
- **covb\***: estimated covariance matrix of *b* of the *ith* equation.
- **y\***: vector of endogenous variable (response values) of the *ith* equation.
- **x\***: matrix of regressors (model matrix) of the *ith* equation.
- **data\***: data frame (including instruments) of the *ith* equation.
- **fitted\***: vector of fitted values of the *ith* equation.
- **residuals\***: vector of residuals of the *ith* equation.
- **ssr\***: sum of squared residuals of the *ith* equation.
- **mse\***: estimated variance of the residuals (mean of squared errors) of the *ith* equation.

- **s2\***: estimated variance of the residuals ( $\hat{\sigma}^2$ ) of the *ith* equation.
- **rmse\***: estimated standard error of the residuals (square root of mse) of the *ith* equation.
- **s\***: estimated standard error of the residuals ( $\hat{\sigma}$ ) of the *ith* equation.
- **r2\***: R-squared (coefficient of determination).
- **adjr2\***: adjusted R-squared value.
- **inst\***: instruments of the *ith* equation.
- **h\***: matrix of instrumental variables of the *ith* equation.

### How to Cite the *twosls* Zelig model

Ferdinand Alimadhi, Ying Lu, and Elena Villalon. 2007. “twosls: Two Stage Least Squares,” in Kosuke Imai, Gary King, and Olivia Lau, “Zelig: Everyone’s Statistical Software,” <http://gking.harvard.edu/zelig>.

To cite Zelig as a whole, please reference these two sources:

Kosuke Imai, Gary King, and Olivia Lau. 2007. “Zelig: Everyone’s Statistical Software,” <http://GKing.harvard.edu/zelig>.

Imai, Kosuke, Gary King, and Olivia Lau. (2008). “Toward A Common Framework for Statistical Analysis and Development.” *Journal of Computational and Graphical Statistics*, Vol. 17, No. 4 (December), pp. 892-913.

### See also

The *twosls* function is adapted from the **systemfit** library by Jeff Hamann and Arne Henningsen [? ].

### References